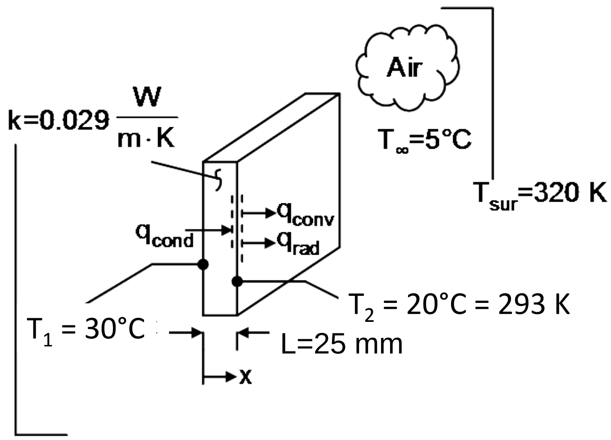
PROBLEM 1.56

KNOWN: Thermal conductivity, thickness and temperature difference across a sheet of rigid extruded insulation. Cold wall temperature, surroundings temperature, ambient temperature and emissivity.

FIND: (a) The value of the convection heat transfer coefficient on the cold wall side in units of $W/m^2\cdot°C$ or $W/m^2\cdot K$, and, (b) The cold wall surface temperature for emissivities over the range $0.05 \le \varepsilon \le 0.95$ for a hot wall temperature of T_1 = 30 °C.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (c) Constant properties, (4) Large surroundings.

ANALYSIS:
(a) An energy balance on the control surface shown in the schematic yields

$$\dot{E}_{in} = \dot{E}_{out} \quad \text{or} \quad q_{cond} = q_{conv} + q_{rad}$$

Substituting from Fourier's law, Newton's law of cooling, and Eq. 1.7 yields

$$k\frac{T_1 - T_2}{L} = h(T_2 - T_\infty) + \varepsilon\sigma(T_2^4 - T_{sur}^4) \qquad (1)$$

or

$$h = \frac{1}{(T_2 - T_\infty)}\left[k\frac{T_1 - T_2}{L} - \varepsilon\sigma(T_2^4 - T_{sur}^4)\right]$$

Substituting values,

$$h = \frac{1}{(20 - 5)^{o}C}\left[0.029\frac{W}{m\cdot K} \times \frac{(30 - 20)^{o}C}{0.02\ m} - 0.95 \times 5.67 \times 10^{-8}\frac{W}{m^2\cdot K^4}(293^4 - 320^4)\ K^4\right]$$

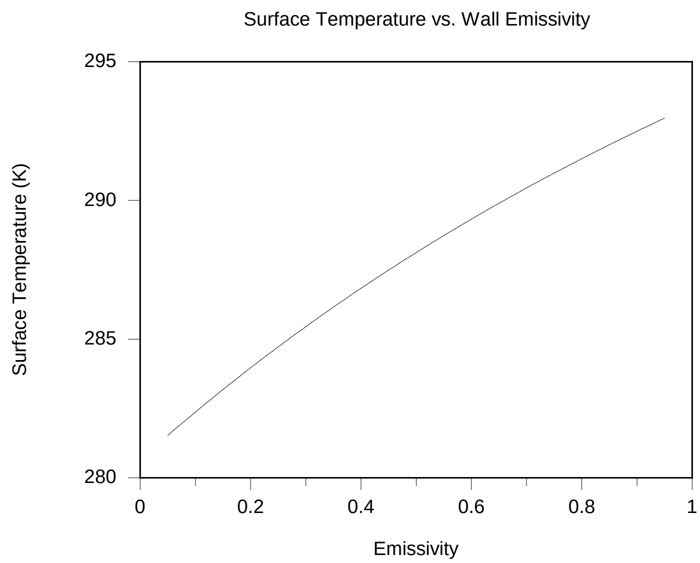
$$h = 12.2\frac{W}{m^2\cdot K} \qquad <$$

Since temperature differences of 1 K and 1°C are equal, $h = 12.2\dfrac{W}{m^2\cdot °C}$. $\qquad <$

Continued...

PROBLEM 1.56 (Cont.)

(b) Equation (1) may be solved iteratively to find T_2 for any emissivity ε. *IHT* was used for this purpose, yielding the following.



COMMENTS: (1) Note that as the wall emissivity increases, the surface temperature increases since the surroundings temperature is relatively hot. (2) The *IHT* code used in part (b) is shown below. (3) It is a good habit to work in temperature units of kelvins when radiation heat transfer is included in the solution of the problem.

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//Problem 1.56

h = 12.2  //W/m^2·K (convection coefficient)
L = 0.02  //m (sheet thickness)
k = 0.029 //W/m·K (thermal conductivity)
T1 = 30 + 273 //K (hot wall temperature)
Tsur = 320 //K (surroundings temperature)
sigma = 5.67*10^-8 //W/m^2·K^4 (Stefan -Boltzmann constant)
Tinf = 5 + 273 //K (ambient temperature)
e = 0.95 //emissivity

//Equation (1) is

k*(T1-T2)/L = h*(T2-Tinf) + e*sigma*(T2^4 - Tsur^4)
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